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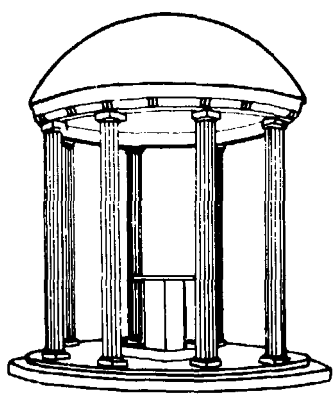

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OPERATIONS RESEARCH AND SYSTEMS ANALYSIS

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AN INVENTORY MODEL FOR SPECIAL HANDLING
OF EXTREME VALUE DEMANDS

Technical Report #19

Douglas Blazer and Marilyn McClelland

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Decision Control Models in Operations Research

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We develop a stationary, discrete-time inventory model with a point τ, such that any demands that exceed the value of τ are not filled from existing stock. We show under what conditions it is economically advantageous to use this model and provide some numerical results. We then show how to find the optimal value for τ.

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FOREWORD

As a part of the on-going program in "Decision Control Models in Operations Research," Mr. Douglas Blazer and Ms. Marilyn McClelland have made an initial study of the economic impact of removing large demands in the determination of an inventory replenishment policy. They treat the multi-period case with linear costs and fixed leadtime; a set-up cost is not included in this paper. Mr. Blazer and Ms. McClelland derive an optimal single-number critical policy when large demand is filtered out. They show how to determine optimally the filter level itself. The paper provides a simple computing algorithm when the demand distribution is negative exponential. Other related reports dealing with this research program are given on the following pages.

Harvey M. Wagner
Principal Investigator

Richard Ehrhardt
Co-Principal Investigator

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Estey, A. S. and R. L. Kaufman (1975), Multi-Item Inventory System Policies Using Statistical Estimates: Negative Binomial Demands (Variance/Mean = 9), ONR and ARO Technical Report 3, September 1975, School of Organization and Management, Yale University, 85 pp.

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Schultz, C. R., R. Ehrhardt, and A. MacCormick (1977), Forecasting Operating Characteristics of (s,S) Inventory Systems, ONR and ARO Technical Report 13, December 1977, School of Business Administration and Curriculum in Operations Research and Systems Analysis, University of North Carolina at Chapel Hill, 47 pp.

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Mosier, C. (1981), Revised (s,S) Power Approximation, Technical Report 18, February 1981, School of Business Administration, University of North Carolina at Chapel Hill, 18 pp.

AN INVENTORY MODEL FOR SPECIAL HANDLING OF EXTREME VALUE DEMANDS

Douglas J. Blazer and Marilyn K. McClelland[+]

-Abstract-

We develop a stationary, discrete-time inventory model with a point τ, such that any demands that exceed the value of τ are not filled from existing stock. We show under what conditions it is economically advantageous to use this model and provide some numerical results. We then show how to find an optimal value for τ.

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We must thank two people whose time, inspiration, and insight were invaluable in the completion of this research. First, special thanks to Professor Richard Ehrhardt whose patience in teaching us his mathematical wizardry was greatly appreciated, and, of course, to Professor Harvey Wagner whose insight lead to this research and whose patience, support, and enthusiasm carried us through to its completion.

TABLE OF CONTENTS

1. INTRODUCTION

Usually stochastic inventory models postulate a probability distribution of demand that makes no qualitative distinction regarding the magnitude of demand. But in real life, a relatively large demand may be attributable to an identifiable type of customer or peculiar event. In such circumstances, it may be possible to anticipate (better forecast) the demand or to handle the demand specially when it occurs. Further, sometimes a customer that demands a large quantity is unwilling to accept a partial shipment when insufficient supply is on hand to fill the entire order. In this paper we analyze when it is economically advantageous to treat large demands (extreme values) differently from others. We formulate a model in which we need not satisfy relatively large demands out of the stock held to meet smaller demands.

1.1 The Model

To simplify the analysis in this initial investigation of the topic, we assume a stationary, discrete-time model, where $\Phi(q)$ is the distribution function of demand q in each period (say, a week), and demand is independently distributed among periods. Let τ designate the smallest value of demand that we consider to be extreme. Then $1 - \Phi(\tau)$ is the probability that we observe a large demand during a period. (In a real situation, more than one customer may place an order in a period, so that q would be the corresponding sum of all customer orders. Then it would be more

appropriate to define an extreme value in terms of an individual customer order. In this paper, however, we avoid the additional complication.)

We examine below the familiar single-critical-number policy model with stationary linear costs. We consider the case of an infinite time horizon; by a trivial alteration of the value of the discount factor, we can extend the analysis to the newsboy problem (single time period).

Let c be the unit cost of each item purchased from a vendor at the start of a period. Assume there is no initial inventory in the first period. Let h be the holding cost per unit of each item in inventory at the end of a period, and π be the penalty cost per unit of each item of unsatisfied demand at the end of a period. We postulate complete backlogging of unfilled regular demand. Assume that the replenishment leadtime is 0 and that the one-period discount factor is $\alpha \le 1$. Suppose that the value of τ is specified. Later we examine how to determine τ to minimize expected total cost per period, where total cost then includes the expense due to specially handling at extreme demand.

It is well known [1] that an optimal critical number S can be found by minimizing

$$L(y) \equiv (1-\alpha)cy + \int_0^y h(y-q)d\Phi(q) + \int_y^\tau \pi(q-y)d\Phi(q) + \int_\tau^\infty hy d\Phi(q) \qquad (1)$$

with respect to y. Note that the last integral in (1) represents the expected cost when demand exceeds τ and therefore is not netted out of y. We assume that in minimizing $L(y)$ we only

need to consider $y < \tau$, since it would be uneconomical to stock more than the largest value that will be filled from that stock. Minimizing $L(y)$ with respect to y yields an optimal stock level $y = S$, where $S \geq 0$ is the smallest number such that

$$\Phi(S) + [1 - \Phi(\tau)] \geq R \tag{2}$$

and

$$R = \frac{\pi - (1-\alpha)c}{\pi + h} . \tag{3}$$

Thus given a value of τ, an optimal critical number is analogous to that for the usual infinite horizon model [2], where here an extreme value for demand is treated as if it were 0.

In Section 2.1 we modify the formulation by adding the costs associated with treating an extreme demand specially. But before doing so, we examine a necessary and sufficient condition for realizing a reduction in expected holding and penalty costs when τ is such that $\Phi(\tau) < 1$, that is, when large demands are removed and treated separately. We examine when the decrease in expected penalty cost is greater than the increase in expected holding cost arising from the special treatment of extreme value demand.

Let S_0 be the smallest number such that

$$\Phi(S_0) \geq R , \tag{4}$$

so that S_0 is an optimal critical number when extreme values are not removed. Then the <u>change</u> in costs due to eliminating extreme value demand is

$$(1-\alpha)c(S_0-S) + h\left[\int_0^{S_0}(S_0-q)d\Phi(q) - \int_0^{S}(S-q)d\Phi(q) - \int_\tau^{\infty}Sd\Phi(q)\right]$$

$$+ \pi\left[\int_{S_0}^{\infty}(q-S_0)d\Phi(q) - \int_S^{\tau}(q-S)d\Phi(q)\right] . \quad (5)$$

For the case where (2) is a strict equality, we have

$$\int_0^{S_0}(S_0-q)d\Phi(q) - \int_0^{S}(S-q)d\Phi(q) - \int_\tau^{\infty}Sd\Phi(q) =$$

$$S_0\int_0^{S_0}d\Phi(q) - \int_0^{S_0}qd\Phi(q) - S\left[\int_0^{S}d\Phi(q) + \int_\tau^{\infty}d\Phi(q)\right] + \int_0^{S}qd\Phi(q) , \quad (6)$$

and, from (2) and (4), we can reduce (6) to

$$R(S_0-S) - \int_S^{S_0}qd\Phi(q) . \quad (7)$$

Similarly, we have

$$\int_{S_0}^{\infty}(q-S_0)d\Phi(q) - \int_S^{\tau}(q-S)d\Phi(q) =$$

$$-S_0\int_{S_0}^{\infty}d\Phi(q) + \int_{S_0}^{\infty}qd\Phi(q) - \left[-S\int_S^{\tau}d\Phi(q) + \int_S^{\tau}qd\Phi(q)\right] , \quad (8)$$

and from (2) and (4)

$$\int_{S_0}^{\infty}d\Phi(q) = \int_S^{\tau}d\Phi(q) = 1 - R . \quad (9)$$

Then (8) becomes

$$(S-S_0)(1-R) - \int_S^{S_0} q d\Phi(q) + \int_\tau^\infty q d\Phi(q) , \qquad (10)$$

and, substituting (7) and (10) into (5), we get that the change in costs is

$$-(h+\pi) \int_S^{S_0} q d\Phi(q) + \pi \int_\tau^\infty q d\Phi(q) . \qquad (11)$$

Thus expected holding and penalty costs per period decrease due to the elimination of extreme demand if and only if (11) is greater than 0, that is,

$$\frac{\pi}{\pi+h} > \frac{\int_S^{S_0} q d\Phi(q)}{\int_\tau^\infty q d\Phi(q)} . \qquad (12)$$

1.2 Numerical Results

Here we illustrate examples where savings in expected holding and penalty costs occur. Consider the undiscounted model where $\alpha = 1$. We calculate $L(y)$ in (1) for $y = S$ and $y = S_0$, as determined by (2) and (4), where we assume that $\Phi(q)$ is either a normal or a uniform distribution. We vary the variance-to-mean ratio and the mean demand to demonstrate conditions where specially treating extreme value demand reduces expected holding and penalty costs. We show in Table I the percentage reduction or increase in expected cost. We let $R = .80$, so that the ratio of π to h

TABLE I

$$\frac{\text{Total Holding and Penalty Costs with Special Handling}}{\text{Total Holding and Penalty Costs without Special Handling}} \times 100\%$$

Assume $R = .8$, $\Phi(\tau) \approx .85$, $\alpha = 1$

Distribution	Variance-to-Mean Ratio	Mean Demand		
		10	20	30
Normal	1	97%	112%	122%
	3		95%	100%
	5			93%
Uniform	1	108%	126%	174%

is 4:1. Also, we let τ be such that $1 - \Phi(\tau) \approx .15$. Table I shows that costs decrease as mean demand gets sufficiently small, and the variance-to-mean ratio gets sufficiently large. Observe from (12) that special handling of extreme value demands also saves costs when R is sufficiently large.

2. DETERMINATION OF OPTIMAL POLICIES

In this section, we add to (1) a special penalty and a special order cost attributable to the extreme-value demands. We then determine values of τ and a stock level that minimize total expected costs. We show a method for finding optimal values of τ and a stock level for the negative exponential distribution.

2.1 Special Penalty and Special Order Cost

Let π' be a special penalty cost per unit of extreme value demand and k' be a fixed special order cost. The expected cost function analogous to (1) becomes

$$L^*(y,\tau) = (1-\alpha)cy + \int_0^y h(y-q)d\Phi(q) + \int_\tau^\infty hyd\Phi(q)$$

$$+ \int_y^\tau \pi(q-y)d\Phi(q) + \int_\tau^\infty \pi' qd\Phi(q) + \int_\tau^\infty k'd\Phi(q) . \qquad (13)$$

Note that if $\pi' = \pi$, then (13) becomes

$$L^*(y,\tau) = (1-\alpha)cy + \int_0^y h(y-q)d\Phi(q) + \int_y^\infty \pi(q-y)d\Phi(q)$$

$$+ \left[\int_\tau^\infty \pi yd\Phi(q) + \int_\tau^\infty hyd\Phi(q) + \int_\tau^\infty k'd\Phi(q)\right] , \qquad (14)$$

which is the expected cost model without special handling of extreme value demands plus an additional cost in the bracketed terms that decreases as τ approaches infinity. Thus for $\pi' \geq \pi$, it is not economically advantageous to special handle extreme value demands.

Now let S_τ be a value for y that minimizes $L^*(y,\tau)$ for a fixed value of τ. Then for an optimum y for a given τ, we denote the expected cost in (13) as

$$L^*(S_\tau,\tau) \equiv G(\tau) \; . \tag{15}$$

An optimum value of τ is found by setting $G'(\tau) = 0$.

Note from (2) that S_τ is the optimal stock level for a value τ, where $S_\tau \geq 0$ is the smallest number such that

$$\Phi(S_\tau) \geq \Phi(\tau) - \frac{h + (1-\alpha)c}{\pi + h} \; . \tag{16}$$

Differentiating (16) as a strict equality with respect to τ yields

$$\rho(S_\tau)S_\tau' = \rho(\tau) \; , \tag{17}$$

where $\rho(\cdot)$ is the probability density of demand. Minimizing $G(\tau)$ with respect to τ and using (16) yields

$$\tau = \frac{(h+\pi)S_\tau + k'}{\pi - \pi'} \quad \text{for} \quad \pi > \pi' \; . \tag{18}$$

Equations (16) as a strict equality and (18) can be solved numerically to yield a value for τ and S_τ that minimize total expected costs.

2.2 Example of Finding an Optimal τ

We now show an algorithm for finding optimal values of τ and S_τ where $\Phi(q)$ is a negative exponential distribution, $\alpha = 1$, and only the special penalty cost π' for extreme value demands is included. From (2) as an equality and (18), we have

$$\tau = \Phi^{-1}\left[\Phi(S_\tau) + \frac{h}{h+\pi}\right] \tag{19}$$

$$\tau = \frac{(h+\pi)S_\tau}{\pi-\pi'} . \tag{20}$$

We let

$$P = \frac{\pi-\pi'}{\pi+h} \tag{21}$$

where P is strictly greater than 0 from (14) and let λ be the parameter in the negative exponential distribution. Then, substituting (20) into (19) we get

$$\tau = -\frac{1}{\lambda}\ln\left(e^{-P\tau\lambda} + \frac{h}{h+\pi}\right) . \tag{22}$$

Starting at $\tau = 1/\lambda$ on the right of (22), successive values for τ are computed until we find a value τ^* that solves (22). The expected cost for τ^* is

$$\left[e^{-\lambda S_{\tau^*}}(S_{\tau^*}+1/\lambda)(h+\pi)\right] - \left[e^{-\lambda\tau^*}(\tau^*+1/\lambda)(\pi-\pi')\right] - h/\lambda . \tag{23}$$

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2. Veinott, Arthur F. (1966), "The Status of Mathematical Inventory Theory," *Management Science*, Vol. 12, p. 745.

SUPPLEMENTARY

INFORMATION

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ERRATUM

AN INVENTORY MODEL FOR SPECIAL HANDLING OF EXTREME VALUE DEMANDS

Technical Report #19

Douglas Blazer and Marilyn McClelland

December 1981

Equation (22) appearing on page 9 should read as follows:

$$\tau = -\frac{1}{\lambda}\ln\left(e^{-P\tau\lambda} - \frac{h}{h+\pi}\right).$$